Chapter 2

The Physical Layer

The Theoretical Basis for Data Communication

- Fourier Analysis
- Bandwidth-Limited Signals
- Maximum Data Rate of a Channel

Fourier Decomposition

- Any reasonable behaved periodic function g(t), with period T can be constructed by summing a number of sines and cosines

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

Fourier Transform

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

- The coefficients can be computed by multiplying both sides with sine and cosine and performing the integral since

$$\int_0^T \sin(2\pi kft) * \sin(2\pi nft)\,dt = \begin{cases} 0 \ldots\ldots & k \neq n \\ \frac{T}{2} \ldots\ldots & k = n \end{cases}$$

The coefficients of the Fourier Series

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

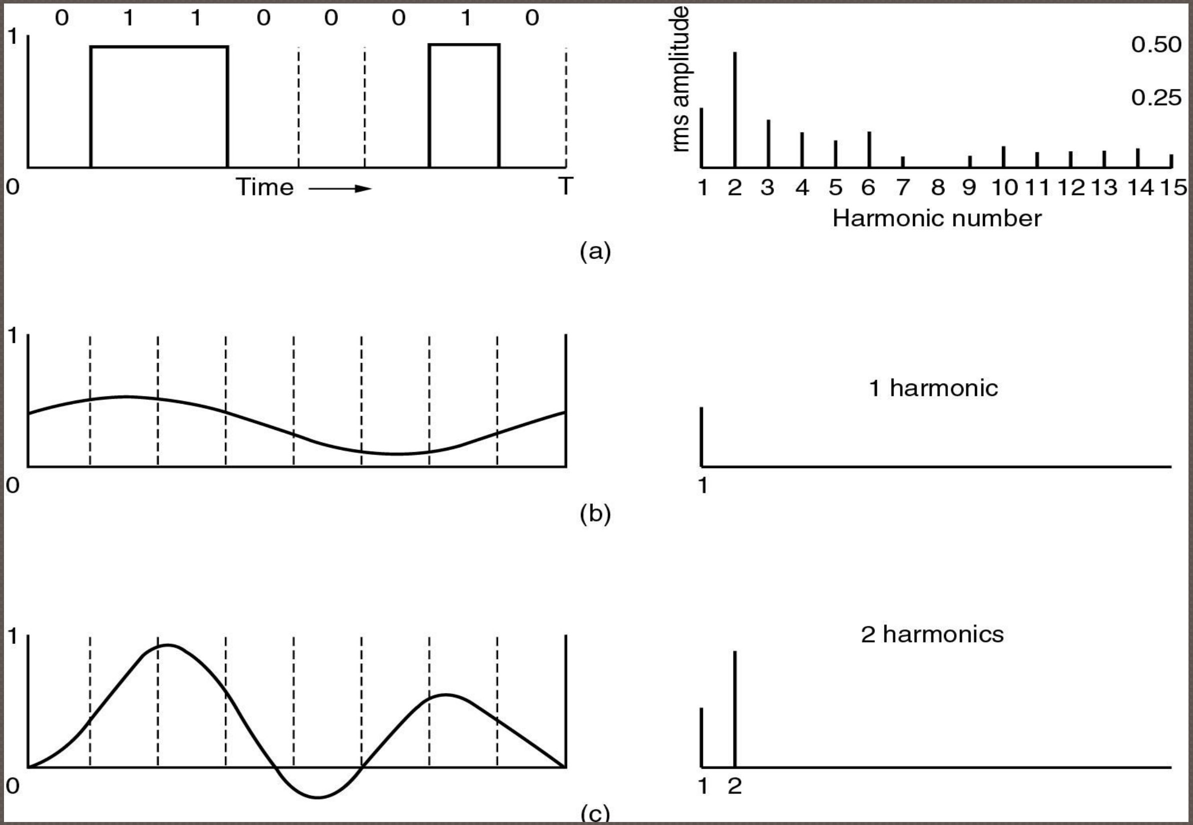
$$f = \frac{1}{T}$$

$$c = \frac{2}{T}\int_0^T g(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T g(t)\sin(2\pi nft)\,dt$$

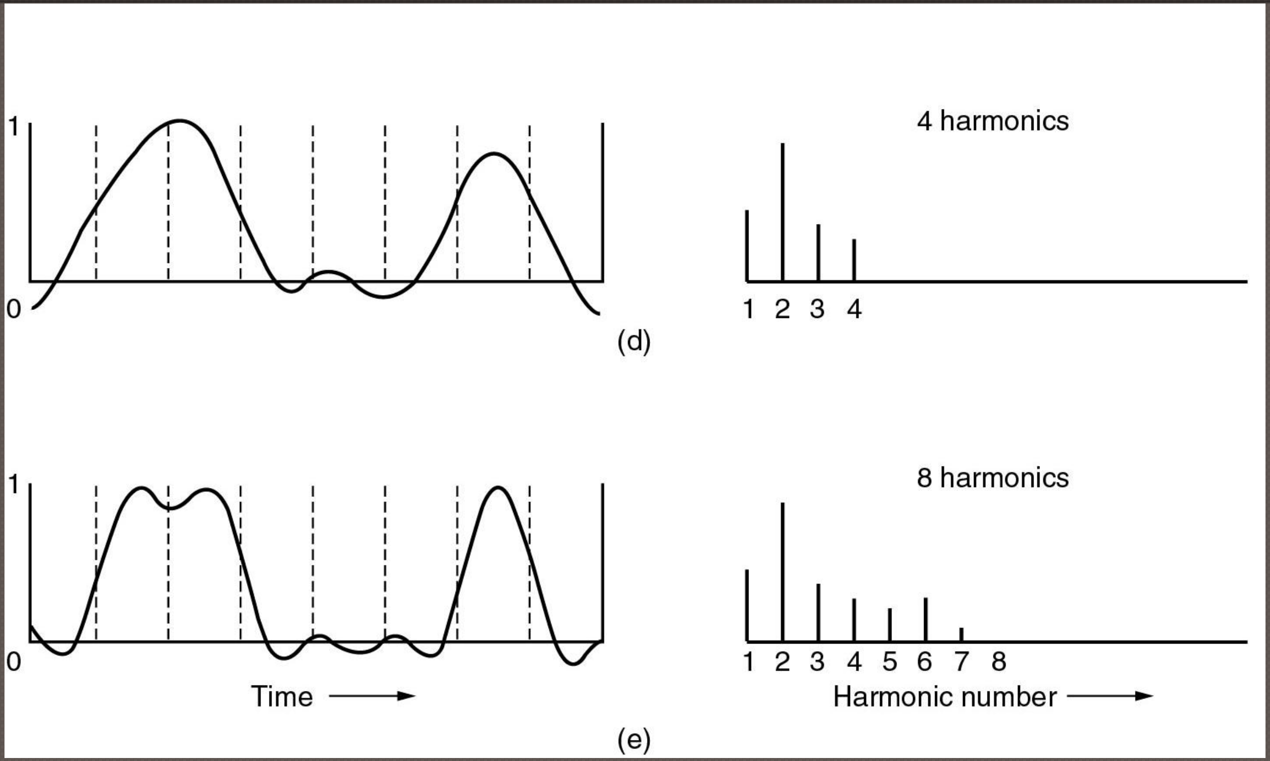
$$b_n = \frac{2}{T}\int_0^T g(t)\cos(2\pi nft)\,dt$$

Bandwidth-Limited Signals



A binary signal and its root-mean-square Fourier amplitudes.
(b) – (c) Successive approximations to the original signal.

Bandwidth-Limited Signals (2)



(d) – (e) Successive approximations to the original signal.

Bandwidth

- Assume a bit rate of *b* bits/sec
- What is the time to send 8 bits 1 bit at a time?
  - *Answer: b*/8 Hz

- Ordinary *voice-grade line* (telephone line) has a cut-off frequency of 3000 Hz
  - The number of the highest harmonic passed through is roughly $3000/(b/8) = 24{,}000/b$

Bandwidth-Limited Signals (3)

Relation between data rate and harmonics.

Data Rate T=8*1/bps Hz=1/T Via 3000 Hz channel

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Maximum Data Rate of a Channel

- Even a perfect channel has a finite transmission capacity!
- Nyquist's Theorem
  - a signal passed through a low-pass filter of bandwidth H can be reconstructed by making only 2H samples per second
  - Sampling more is pointless since the higher frequency components that could be recovered, have already been filtered.

Nyquist's Theorem

- For a H Hz channel and a low-pass channel with cutoff at H Hz, if a signal consists of V discrete levels, then the maximum data rate is

$$D = 2H \log_2 V \frac{bits}{\sec}$$

- Example: a noiseless 4-kHz channel cannot transmit binary (i.e. 2 level) signals at a rate exciding 8000bps

Shannon's Theorem

- Maximum data rate for a noisy channel whose bandwidth is H Hz, and whose signal to noise ratio is S/N, is:

$$M = H \log_2 (1 + \frac{S}{N}) \quad bits/\sec$$

- S/N is expressed as Decibels (db)
  - A ratio of 10, i.e. S/N =10 it means 10db
  - A ratio of 100 is 20 db
  - A ratio of 1000 is 30 db

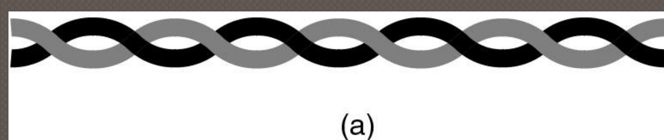
Example of Shannon's Theorem

- According to Shannon, what is the maximum data rata is we use a noiseless 4-kHz channel which has 30db signal to noise ratio?
- Answer:
  - H=4,000
  - 30 db means that S/N=1000
  - M=4,000 * $\log_2$ 1001 = 40,000 bps

Guided Transmission Data

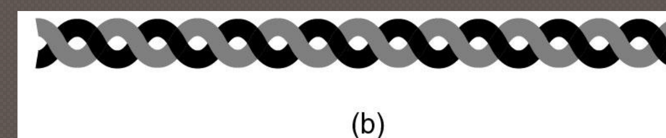
- Magnetic Media
- Twisted Pair

16 MHz bandwidth



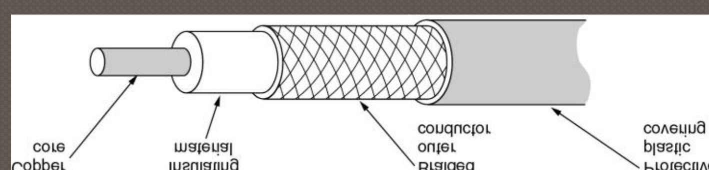
(a) Category 3 UTP.

100 MHz bandwidth



(b) Category 5 UTP.

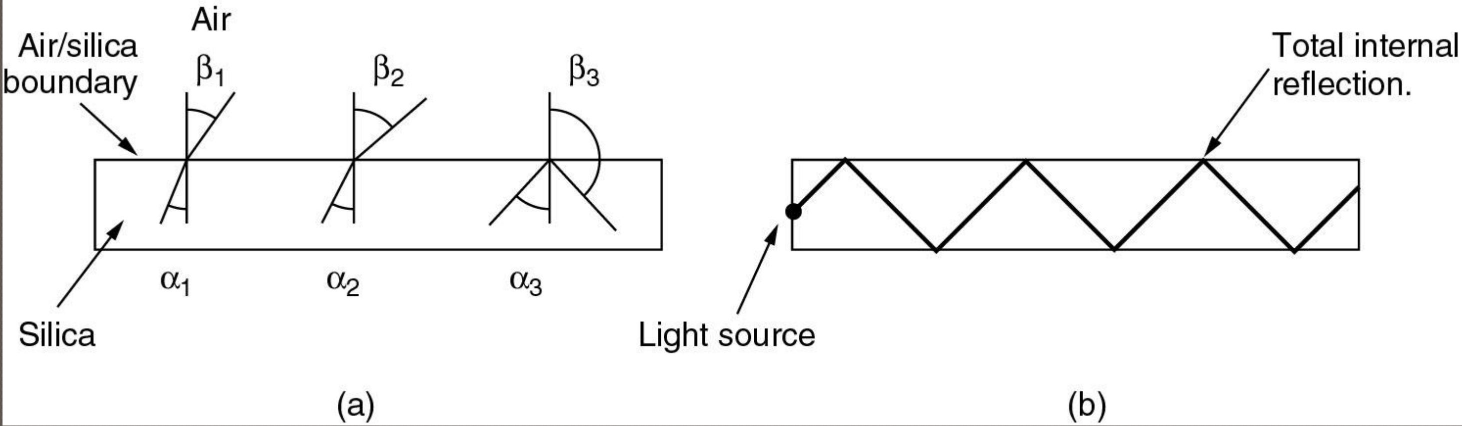
- Coaxial Cable



1 GHz bandwidth

- Fiber Optics

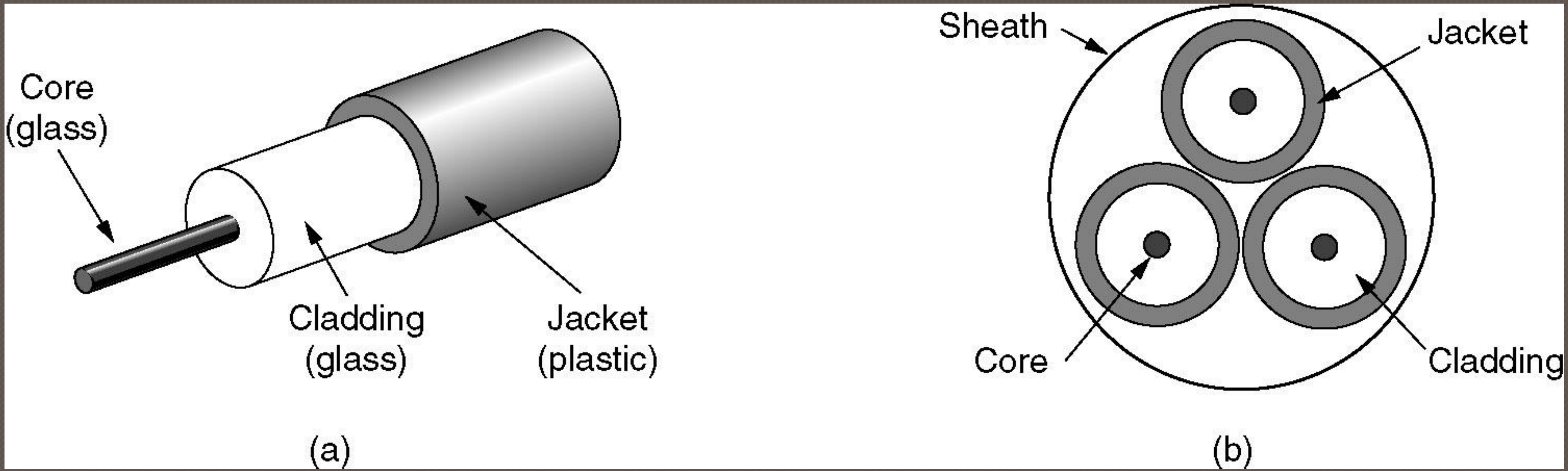
Fiber Optics



(a) Three examples of a light ray from inside a silica fiber impinging on the air/silica boundary at different angles.
(b) Light trapped by total internal reflection

- Single mode fiber transmit at 50 Gbps for 100km without amplification.

Fiber Cables



(a) Side view of a single fiber.
(b) End view of a sheath with three fibers.

Fiber Cables (2)

- Two source of light: LED and semiconductor lasers

Item	LED	Semiconductor laser
Data rate	Low	High
Fiber type	Multimode	Multimode or single mode
Distance	Short	Long
Lifetime	Long life	Short life
Temperature sensitivity	Minor	Substantial
Cost	Low cost	Expensive

A comparison of semiconductor diodes and LEDs as light sources.